SI : Math 166

Tests on Series : Section (9-2, 9-3, 9-4)

(i) Integral Test : $\sum_{K=1}^{\infty} a_K$ series is positive and decreasing

ie $a_K > a_{K+1} > a_{K+2}$

then take $a_K = f(K)$ make a_K a function of x and integrate

$\int_1^{\infty} f(x)\,dx \rightarrow$ converges than $\sum_{K=1}^{\infty} a_K$ also converges

(ii) ordinary Comparison : we have to series. $\sum a_n$ and $\sum b_n$.

and $a_n \le b_n$. (a) if $\sum b_n$ converges then $\sum a_n$ converges too

$\because \sum b_n$ is greater than $\sum a_n$.

but if $\sum b_n$ is diverging we cannot say any thing about $\sum a_n$

~~(b) $a_n \le b_n$:~~

$a_n \le b_n$. (b.) if $\sum a_n$ is diverging than $\sum b_n$ is also diverging $\because$ the smaller (an) diverges than bigger (bn) should diverge too.

but if $\sum a_n$ is converging than we cannot say any thing about $\sum b_n$.

(iii) <u>Limit Comparison Test:</u> we got two series $\sum a_n$ and $\sum b_n$. $\sum a_n$ is given in the question

and $\sum b_n$ is a series similar to $\sum a_n$ but we know if it diverges ~~diverges~~ or converges..

~~than~~ then find $\lim_{n \to \infty} \frac{a_n}{b_n} = L$

then if $0 < L < \infty$ ie a finite number but not 0

Then $\sum a_n$ and $\sum b_n$ converge and diverge TOGETHER!

<u>ie.</u> if $\sum b_n$ converges -- $\sum a_n$ also converges.

If $\sum b_n$ diverges -- $\sum a_n$ also diverges.

If $L = 0$ then test is <u>in-conclusive!</u>

~~(Use another test then~~

(IV)

<u>Ratio Test:</u> $\sum a_n$ is a positive terms series.

then $\rho = \lim_{n \to \infty} \frac{a_{n+1}}{a_n}$

If (a) $\rho < 1$ converges
(b) $\rho > 1$ or ∞ series diverges
(c) $\rho = 1$ test is <u>inconclusive!</u>

<u>ex:</u> $\sum a_n = \sum \frac{1}{2^n}$

then $a_n = \frac{1}{2^n}$ and $a_{n+1} = \frac{1}{2^{n+1}}$

$\rho = \dfrac{\frac{1}{2^{n+1}}}{\frac{1}{2^n}} = \frac{1}{2} < 1.$

Converges!